Optimised Consensus for Highway Speed Limits via Intelligent Speed Advisory Systems

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***Abstract*—In this paper, an application based on the Intelligent Speed Adaptation (ISA) system is proposed to reduce CO_2 emissions of vehicles running on the highway. We apply the idea of optimised consensus to solve the emission optimisation problem in a simplified highway scenario through simulation study. Our approach shows that total CO_2 emissions of vehicles can be minimised if all vehicles follow the reference speed signal derived from the ISA.**

I. Introduction

Intelligent Speed Adaptation (ISA), is designed as a road monitoring and advising system that is capable of informing the speed limit information to the drivers on the road [1]. The speed information can be used to affect drivers' behaviour for improving traffic efficiency and safety. In this system, drivers can get the speed information constantly from the monitor installed on their vehicles and take actions when necessary. In the meanwhile, a wide variety of applications based on Vehicular Ad-hoc Network (VANET) have been investigated and developed providing fundamental capabilities for Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communication to be deployed in the framework of Intelligent Smart Transportation System (ITS).

Motor traffic has been recognised as one of the major sources for pollutants generation in urban cities [2]. Although Electric Vehicles (EV) based "green" applications have been proposed for many years for pollutant reductions, there is still a large share of conventional fossil fuel based vehicles running on the road, which produce harmful emissions to the environment. In order to mitigate the pollution level, one of the most effective methods is to influence the driving behaviours of the vehicles such that some common global goal can be achieved (e.g. minimisation of total CO_2 emission). Nowadays, with various communication infrastructure being developed, such as V2V and V2I, vehicles can cooperate effectively in the constructed network to achieve the desired system performance while satisfying some local constraints. Such applications can be well fitted in the cooperative control framework.

In this paper, our objective is to design an ISA system that can broadcast some essential messages to all vehicles suggesting that if all vehicles can follow the same reference speed signal, the total emission can be minimised. This problem can be formulated as an optimisation problem subject to some local consensus constraints. Our idea is to apply a decentralised consensus algorithm for sharing the speed information of the vehicles through the road communication infrastructures so that the overall objective optimisation can be achieved.

II. System Model

Let us consider a scenario in which a number of vehicles are driving along the given stretch of highway on different lanes in the same direction. Let n denote the total number of vehicles on a particular section of the highway where the ISA broadcast signal can be received. Each vehicle equipped with a specific communication device, which is able to receive and transmit messages to either vehicles or the road infrastructure nearby, is regarded as a mobility agent. Here we define the set $I := \{1, 2, ..., n\}$ for indexing the agents. Let $\mathbf{1}$ denote the vector with all entries equal to 1 of appropriate length. Let $s_i(t)$ denote the recommended speed of the i^{th} agent at time slot t. The corresponding recommended speed vector for all vehicles at time t is given by $\mathbf{s}(t)^{\mathrm{T}} := [s_1(t), s_2(t), ..., s_n(t)]$. In addition, each agent is associated with a CO_2 emission utility function $f_i : \mathbb{R} \mapsto \mathbb{R}$, which we assume to be convex, continuous and second order differentiable. We also assume that each agent can adjust $s_i(t)$ based on the knowledge of $f_i(t)$. The first derivative of the i^{th} utility function f_i is denoted as $f_i' : \mathbb{R} \mapsto \mathbb{R}$. Specifically, we shall find the optimal solutions $\mathbf{s}(t)$ for the following minimisation problem.

$$\begin{aligned} \underset{\mathbf{s}(t)}{\text{minimise}} \quad & \sum_{i=1}^{n} f_i(s_i(t)) \\ \text{subject to: } & s_i(t) = s_j(t), \ \forall i \neq j \in I. \end{aligned} \tag{1}$$

In order to solve (1), we recall the lemma in [3] which relates to a consensus algorithm in the case of a common input:

Lemma 1: Let $\{P_k\} \subset \mathbb{R}^{n \times n}$ be a sequence of matrices from a finite set of primitive, row-stochastic matrices with strictly positive main diagonal entries, and ϑ_k be a sequence of real numbers. Then, if $x_k = (x_k^1, \ldots, x_k^n)^{\mathrm{T}}$ evolves for some $x_0 \in \mathbb{R}^n$ according to:

$$x_{k+1} = P_k x_k + \vartheta_k \mathbf{1} \tag{2}$$

then the elements of x_k will approach each other over time, which is:

$$\lim_{k \to \infty} x_k^p - x_k^q = 0, \ \forall p, q \in I \tag{3}$$

According to the basic stationary point mechanism, we note that the optimisation problem described in (1) is equivalent to solve the following Lagrange equations:

$$\frac{\partial\left[\sum_{i=1}^{n} f_i(s_i(t)) + \lambda_i(s_i(t) - s_j(t))\right]}{\partial s_i(t)} = 0,\ \forall i \neq j \in I \quad (4)$$

where λ_i represents the Lagrange multiplier for the i^{th} constraint equation (i.e. $s_i(t) - s_j(t) = 0$).

Then, it can be easily shown from equations (4) that our optimisation problem depicted in (1) correspond to finding the solution $\mathbf{s}(t)$ of the following equations:

$$\begin{gathered}\sum_{i=1}^{n} f_i'(s_i(t)) = 0 \\ s_i(t) = s_j(t),\ \forall i \neq j \in I.\end{gathered} \quad (5)$$

To solve equations (5), we propose the Algorithm 1 below.

Algorithm 1 Decentralised Consensus algorithm

1: **for** each $i \in I$ **do**
2: $g_i(t) = \eta \sum_{j \in N_t^i} (s_j(t) - s_i(t))$
3: $e_i(t) = \sum_{i=1}^{n} f_i'(s_i(t))$
4: $s_i(t+1) = s_i(t) + g_i(t) + \mu \cdot (0 - e_i(t))$
5: **end for**

Comment: In Algorithm 1, N_t^i denotes the set of neighbours of the agent i which can send its recommended speed signal to the i^{th} agent at time t. Moreover, with certain conditions on η and μ, this algorithm is proved to asymptotically converge to a stable fixed point. Such a fixed point is also the optimal solution for the problem (1). More details will be given in [4].

III. Case Study

In this section we evaluate the performance of the algorithm in the case that there are 40 vehicles travelling along a section of the highway. We use the same average-speed model of reference [5] to infer the CO_2 emissions as a function of the speed. Accordingly, the emissions depend on the class of the specific vehicle and we assumed that there were 10 vehicles for each class. Then, the corresponding emission functions can be computed according to the following equation:

$$f_i(s_i(t)) = \frac{a + b \cdot s_i(t) + c \cdot s_i(t)^2 + d \cdot s_i(t)^3}{s_i(t)} \quad (6)$$

where the values of the parameters a, b, c and d for different classes of vehicles are reported in Table I [5]. The corresponding function representations are illustrated in Fig.1.

TABLE I. Emission Factors for different Types of Vehicles

Emission Standard	a	b	c	d
Euro I	3.7473E3	1.9576E2	-8.5270E-1	1.0318E-2
Euro II	3.7473E3	1.8600E2	-8.5270E-1	1.0318E-2
Euro III	3.7473E3	1.6774E2	-8.5270E-1	1.0318E-2
Euro IV	3.7473E3	1.5599E2	-8.5270E-1	1.0318E-2

We assumed that at the beginning of the simulation the initial speeds of the vehicles were randomly distributed between 60 and 80km/h. The recommended speed dynamics of each vehicle are shown in Fig.2. Our result shows that the total CO_2 emissions is minimised if the vehicles are recommended to travel at 74km/h. In this case, we obtain the minimum amount of emitted CO_2, corresponding to 8.8165kg/km.

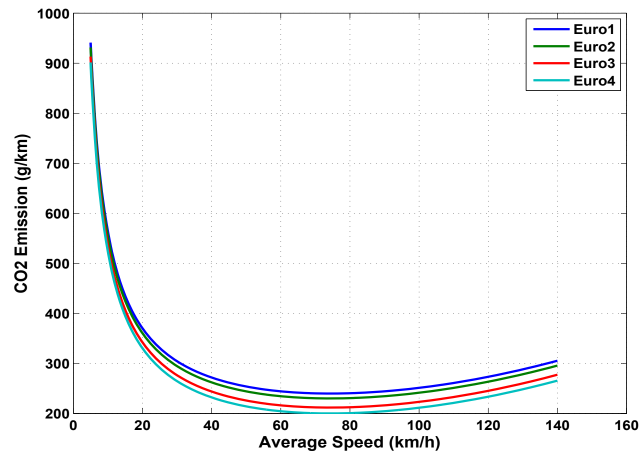

Fig. 1. CO_2 Emission functions for different type of vehicles

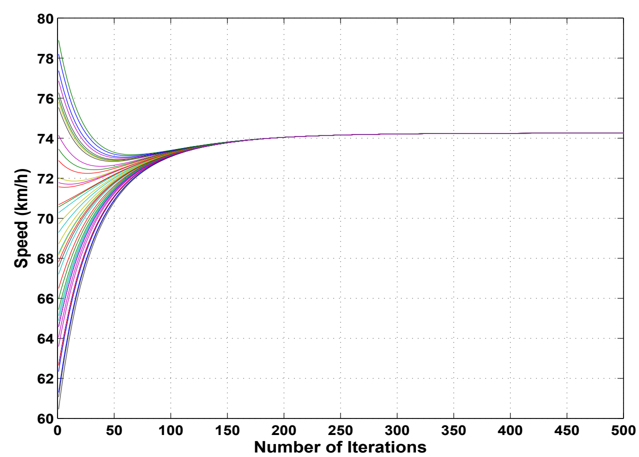

Fig. 2. Recommended speed dynamics for all vehicles considered

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